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MASS ADDITION OF A SUPERSONIC JET

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Presented are results of an experimental determination of the mass addition for a supersonic jet in flow regimes with $n = P_{ch}/P_a \neq 1$. An empirical formula is given relating the mass addition in the initial section to the Mach number M_a at the nozzle exit, the ratio n, and the distance from the nozzle exit section to the section examined.

In many jet problems, interaction of a supersonic jet with a wall or channel inlet is considered. Multiple jet interaction is also of interest, especially if a closed region results. Practical examples are afforded by gas ejectors (especially where there is no ejected flow and the length of the mixing chamber is insufficient for complete flow mixing) and the working sections of wind tunnels. The question of the formation of a "base" pressure in regions bounded by several jets is of particular interest.

In all problems of this type it is desirable to know the so-called mass addition of the jet. A jet of gas, propagating in a stationary or moving medium, interacts with the medium at the jet boundary. Particles at the periphery of the jet collide with stationary particles of the medium and transfer to them part of their energy, thus imparting a velocity directed along the jet axis.

Accordingly, as the distance from the nozzle exit section increases, the mass of air composing the jet proper increases continuously, and at a certain distance from the exit has the value

$$G = G_0 + G_{ad} = G_0 (1 + \bar{q}_{ad}), \quad \bar{q}_{ad} = G_{ad}/G_0. \tag{1}$$

In the case of a known law of velocity and density distribution along the axis and at each cross section of the jet the mass addition may easily be computed theoretically [1].

In the case of a supersonic jet discharging from a nozzle with $n = p_{ch}/p_a$ other than unity, the theoretical determination of the mass addition is made difficult by the complex structure of the jet over the initial section.

The mass addition of the jet G_{ad} at a given cross section obviously depends on its dynamic characteristics (Mach number) and on its surface area between the nozzle exit and the given cross section. At a given distance from the nozzle exit, the surface area is proportional to the ratio n. Thus, for a jet flowing in a stationary medium

$$G_{ad} = f(n, M_a, \bar{X}). \tag{2}$$

Our method of determining G_{ad} was based on the fact that the investigated jet flowed in a chamber (Fig. 1), whose end wall contained an interchangeable ring diaphragm. The size of the aperture d_e in the diaphragm was so chosen that the size of the jet at section b was known to be less than the diameter of the outlet aperture. A jet, propagating in a chamber of length $\bar{X}$, increases in mass by an amount G_{ad} at the expense of the air in the chamber outside the jet. Since the jet does not touch the rim of the outlet aperture, all its mass $G = G_0 + G_{ad}$ leaves the chamber. Because the amount of air in the chamber decreases continuously due to the outflow of additional mass, the pressure there must fall. However, at constant M_a, p_0, and d_e, the pressure in the chamber remains constant. It is evident that to keep p_{ch} constant, air must enter from outside at a rate equal to the mass addition, through the annular gap between the rim of the outlet aperture and the edge of the jet. This has been observed experimentally. Air from the surrounding medium with a stagnation pressure p_{at} is accelerated to a velocity v_b at the ring diaphragm, whereupon its pressure falls to the value p_{ch}. The pressure in the chamber determines, in its turn, the value of the ratio n and, consequently, the size of the jet at section b and the area through which air enters the chamber from the atmosphere. Note that in the gap between the diaphragm and the jet the back flow velocity obviously cannot exceed the speed of sound. If we measure the velocity fields in the annular section, for given p_0, p_{ch} and geometrical dimensions of the chamber and nozzle, knowing the size d_j of the jet, and assuming $\rho_b = \rho_{at}$ (which is evident), we can determine the mass back flow rate, which is clearly equal to the mass addition of the jet. By varying the pressure ahead of the nozzle and the nozzle geometry, we can easily obtain the dependence of G_{ad} on M_a and n.

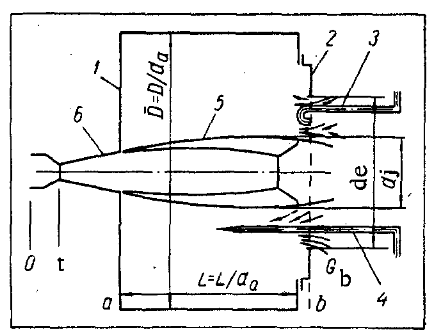

Fig. 1. Test setup: 1) chamber; 2) removable ring diaphragm; 3) total pressure probe; 4) static pressure probe; 5) boundary of jet; 6) nozzle.

The boundary of the main jet, required in determining the area of back flow $F_b = \pi(d_e^2 - d_j^2)/4$, is evaluated by measuring the total pressure field at section b; this was done with an accuracy of 147 N/m^2.

The velocity distribution in the back flow zone was determined from the total and static pressures measured by the corresponding probes, using isentropic flow formulas to obtain the velocity.

The mass back flow rate may easily be obtained from the experimental velocity distribution v(r) at section b:

$$G_b = 2\pi g \rho_{at} \int_{r_j}^{r_b} v(r) r dr.$$

To find G_{ad}, experiments were carried out on models with the following parameters: I—$M_a = 1.0$; $d_t = 8.0$; $d_a = 8.0$ mm; $d_e = 32$ mm; II—2.02; 6.46; 8.47; 32; III — 2.45; 5.14; 8.16; 24; IV—2.85; 4.31; 8.26; 18.

Different d_e values were chosen as M_a was varied, to obtain an annular gap of sufficient size (in comparison with the probe dimensions). The transverse diameter of the models was made equal to $\bar{D} = D/d_a = 6.5$, to avoid large back flow velocities. The model length was varied in the range $\bar{L} = L/d_a = 0$-7. Accuracy of location of the probes was ±0.05 mm with a measuring interval $\Delta r = 0.5$ mm.

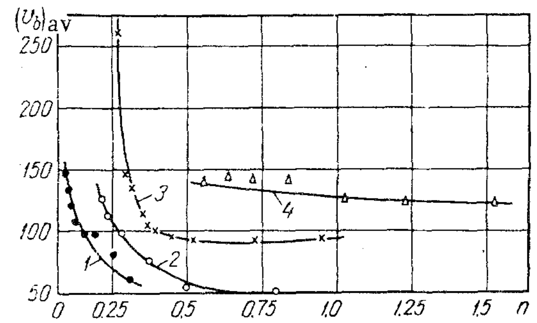

Fig. 2. Velocity fields in main jet and back flow for $\bar{L} = 4.0$ and $\bar{D} = 6.5$: 1) $M_a = 1.0$, $f_b = 16.0$; 2) 2.02, 16.0; 3) 2.45, 9.0; 4) 2.85; 5.06.

The results of the experiments are given in Figs. 2, 3, and 4. As expected, measurements showed that the stagnation pressure in the back flow at section b was constant over the radius and equal to the atmospheric pressure. The stagnation pressure begins to increase at the boundary of the main jet. The results of measurements giving the dependence of the location of the boundary of the main jet, and hence of F_b, on M_a and n, indicate that as n decreases the jet boundary tends to coincide with the rim of the outlet aperture, i.e., F_b tends to zero. Over quite a large range of variation of n the position of the jet boundary is independent of n. When the jet boundary begins to touch the rim of the outlet aperture, n attains a limiting value. No increase of pressure ahead of the nozzle, without change in chamber or nozzle geometry, will further affect the value of n (and hence the position of the jet boundary, the system of shock waves, and the ratio of the pressure ahead of the nozzle p_0 to that in the chamber p_{ch}).

The static pressure p_b in the annular gap $\delta = d_e - d_j$ varies over the radius in a complex manner. It is not possible to measure p_b directly at the rim of the diaphragm because of the finite thickness of the probe, so that the first measurement point was $\Delta r_0 = 0.5$ mm from the rim. The edge pressure has a minimum value, then increases with distance from the rim, and begins to drop again as the boundary of the main jet is approached. At the interface between the main jet and the back flow the variation of p_b is smooth.

The velocity fields in the back flow were calculated from the measured total and static pressures. As an example, Fig. 2 shows the velocity fields for various n. It can be seen from the graphs that the radial variation of v_b corresponds closely to that of static pressure, and that the smaller n becomes, the more the back flow zone contracts, and the higher the velocities.

The average back flow velocities $(v_b)_{av}$ also increase as n decreases (Fig. 3), and when n approaches the limiting value n_l, the average velocity tends to the critical value.

Thus, in this case there are two opposing flows at section b: the main flow with mass flow rate $G = G_0 + G_{ad}$ and a back flow with mass flow rate $G_b = G_{ad}$.

By integrating (3) we obtained values of G_b as a function of n, M_a, and q_{ad}.

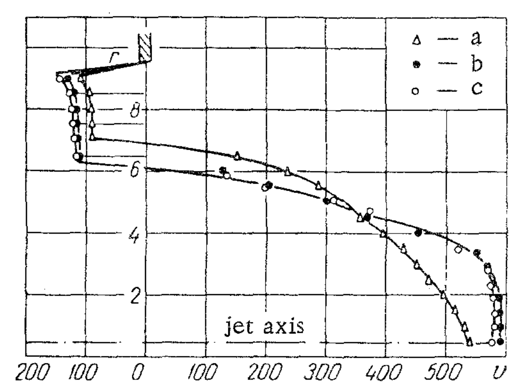

Fig. 3. Variation of average back flow velocity with a) n = 2.04, p_0 = 12; b) 1.23, 20; c) 1.02, 24.

The experimental data (Fig. 4) show that the q_{ad} does not depend on M_a, but only on n. This may be explained by the fact that the basic dynamic characteristic determining mixing is obviously the Mach number at the jet boundary, and this is known to be a function only of the ratio of the pressure ahead of the nozzle to that in the surrounding medium. The scatter of the experimental points at small values of n can be attributed to neglect of the compressibility effect in determining velocities, when the back flow velocity approaches the critical value.

Investigations of the effect of jet length on $\bar{q}_{ad}$ have shown that for any length (within the limits of the initial section) the dependence of $\bar{q}_{ad}$ on n is linear, while the slope of the curves is a function of the distance $\bar{X}$ from the nozzle exit section to the jet section under examination (Fig. 4).

The experimental dependence of $\bar{q}_{ad}$ in the initial section of a supersonic jet with $n \neq 1$ is well approximated by the expression

$$\bar{q}_{at} = 0.0502 n \bar{L}^{1.57}, \tag{4}$$

where $\bar{L} = L/d_a = 0-7$.

Thus, in view of the lack of a theoretical solution, Eq. (4) can be used successfully in many calculations requiring an estimate of the mass flow rate in a supersonic jet at a distance from the nozzle.

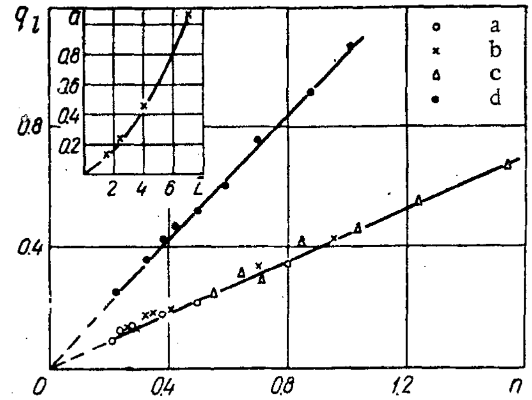

Fig. 4. Dependence of relative mass addition on n ($q_{ad} = an$) with $M_a = 2.02$ and $\bar{L} = 4.0$ a) 2.45 and 4.0 b) 2.85 and 4.0 c) 2.45 and 7.0 and dependence of a on $L(a = 0.0502\,\bar{L}^{1.57})$ with $M_a = 2.45$, $\bar{D} = 6.5$, $f_b = 16$.

NOTATION

M_a – Mach number at nozzle exit; G – mass flow rate at an arbitrary cross section of the jet; G_0 – mass flow rate of gas issuing from nozzle; G_{ad} – quantity of gas added to the jet (between nozzle exit and some specified section) from the surrounding medium; $\bar{q}_{ad}$ – relative mass addition; p_{ch}, p_a, p_0 and p_{at} – pressures in chamber, at nozzle exit, ahead of nozzle, and atmospheric; p_b – static pressure in back flow at section b; ρ_b, v_b – density and velocity in back flow at section b; X – length of jet; d_j, d_e, and d_a – diameters of jet at section b, of outlet aperture and of nozzle exit section; D – transverse dimension of chamber; L – length of chamber.

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